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DEVELOPMENT OF A LAMINAR BOUNDARY LAYER BEHIND A SUCTION POINT

By W. Wuest

Translation of "Entwicklung einer laminaren Grenzschicht hinter einer Absaugestelle." Ingenieur Archiv, Vol. 17, 1949.

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DEVELOPMENT OF A LAMINAR BOUNDARY LAYER

BEHIND A SUCTION POINT*

By W. Wuest

1. INTRODUCTION

Boundary-layer suction originally was applied to reduce the boundary-layer thickness and therewith the inclination to flow separation; however, since the properties of bodies with small drag have been improved more and more, attention was drawn to an increased extent to the reduction of surface friction. One now strived toward keeping the boundary layer laminar as long as possible, thus to defer the transition point to turbulence as far as possible. Boundary-layer suction was recognized to have a favorable effect in this sense, and therewith the velocity distribution in a laminar boundary layer behind a suction point acquired heightened interest. The stability of a laminar velocity profile is very severely affected by the shape of this profile.

In a considerable number of theoretical reports (reference 1) the case of continuous suction was treated for reasons of mathematical simplicity; permeability of the wall surface was assumed. In further reports, the stability of laminar boundary-layer profiles in case of continuous suction was treated and a considerable rise in the stability limit was determined; however, a technical realization of such permeable walls with sufficiently smooth surface and adequate material strength characteristics is difficult. For structural reasons, it is simpler to arrange single-suction slots. In addition to the suction effect proper, there appears here the sink effect first discussed in detail by L. Prandtl and O. Schrenk (reference 2) and recently treated by Pfenniger (reference 3) in an instructive experimental investigation.

Below, the pressure variation along the wall as well as, in particular, the sink effect are disregarded. Figure 1 shows the practical realization of such a case. We assume that on a flat plate A, a laminar boundary layer ("Blasius boundary layer") develops at constant pressure. We assume a second plate B arranged beginning from a certain point x_0 at the distance y_0 parallel to the first plate so that a suction slot

*"Entwicklung einer laminaren Grenzschicht hinter einer Absaugestelle." Ingenieur Archiv, Vol. 17, 1949, pp. 199-206.

is formed between the two plates. The magnitude of the power requirement for suction is assumed to be precisely such that merely the part of the boundary layer situated between the two plates is removed. Thus, there begins above the plate B a new laminar boundary layer which is distinguished from the Blasius boundary layer by another initial condition. The new boundary layer forms at its start the outer part of a Blasius boundary layer.

2. BOUNDARY-LAYER EQUATION AND ASYMPTOTIC BEHAVIOR

By introduction of the stream function and the total pressure, the boundary-layer equation may be transformed by the well-known method (reference 4) into

$$\frac{\partial g}{\partial x} = \nu u \frac{\partial^2 g}{\partial \psi^2} \quad (1)$$

where $g = p + \frac{\rho}{2} u^2$ and $\frac{\partial \psi}{\partial y} = u$. We limit ourselves to the case that the flow takes place outside of the boundary layer at the velocity $u_1 = \text{const.}$, thus to the flat plate and put furthermore

$$g = -\frac{\rho}{2} u_1^2 (1 - q(x, \psi)) + \text{Const.} \quad (2)$$

or, respectively

$$u = u_1 \sqrt{q} \quad (3)$$

This statement has been chosen so that for large ψ -values, q assumes the value 1. Equation (1) is thereby transformed into

$$\frac{\partial q}{\partial x} = \nu u_1 \sqrt{q} \frac{\partial^2 q}{\partial \psi^2} \quad (4)$$

From the definition of the stream function and from equation (3), one further obtains with $\eta = \psi / \sqrt{\nu u_1 x}$

$$\frac{\partial u}{\partial y} = \frac{1}{2} \frac{u_1}{x} \sqrt{\frac{u_1 x}{v}} \frac{\partial q}{\partial \eta} \quad (5)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{2} \frac{u_1}{x^2} \left(\frac{u_1 x}{v} \right) \sqrt{q} \frac{\partial^2 q}{\partial \eta^2} \quad (6)$$

$$y = x \sqrt{\frac{v}{u_1 x}} \int_0^y \frac{1}{\sqrt{q}} d\eta \quad (7)$$

In order to investigate the asymptotic behavior of the differential equation (3), we put for large values of ψ

$$q = 1 - q_w \quad \text{with} \quad q_w \ll 1 \quad (8)$$

In first approximation, one then obtains

$$\frac{\partial q_w}{\partial x} = vu_1 \frac{\partial^2 q_w}{\partial \psi^2} \quad (9)$$

This differential equation, however, is mathematically identical with the differential equation of a nonsteady flow independent of x which has been treated before (reference 5); the time t is now replaced by the stipulated space coordinate x . It also corresponds to the well-known heat-conduction equation. The general solution is therefore given by

$$q_w(\psi, x) = \frac{1}{2} \int_0^\infty q_w(\psi', x_0) \frac{\partial}{\partial \psi'} \left[\Phi \left(\frac{\psi + \psi'}{\sqrt{4v u_1 (x - x_0)}} \right) - \Phi \left(\frac{\psi - \psi'}{\sqrt{4v u_1 (x - x_0)}} \right) \right] d\psi' + \int_{x_0}^x q_w(0, x') \frac{\partial}{\partial x'} \Phi \left(\frac{\psi}{\sqrt{4v u_1 (x - x')}} \right) dx' \quad (10)$$

Therein

$$\Phi = \frac{2}{\sqrt{\pi}} \int_0^z e^{-y^2} dy$$

is the known error integral. W. Tollmien (reference 6) has investigated this solution for two special cases where the first integral disappears. For the boundary layer with suction, however, this will no longer be the case.

3. BLASIUS BOUNDARY LAYER

Although we presupposed that the velocity u_1 at the edge of the boundary layer is constant, the problem of the suction boundary layer to be treated here nevertheless differs from the flow on a simple flat plate ("Blasius boundary layer") by the fact that other initial conditions exist; rather, the Blasius boundary layer is contained as special solution among the suction boundary layers since there $x_0 = 0$, thus suction point and beginning of the plate A (fig. 1) coincide. Since we shall make use of this special solution for the later calculation, we shall first consider the Blasius boundary layer. It is distinguished by the fact that q may be regarded as dependent merely on a quantity $\eta = \psi/\sqrt{u_1 x}$. One then obtains from equation (4) the following differential equation of the Blasius boundary layer

$$\eta \frac{d\sqrt{q_B}}{d\eta} + \frac{d^2 q_B}{d\eta^2} = 0 \quad (11)$$

The solution may be written in the following form

$$q_B = \beta \eta \left(1 + a_1 \left(\frac{\eta^3}{\beta} \right)^{1/2} + a_2 \left(\frac{\eta^3}{\beta} \right)^{2/2} + a_3 \left(\frac{\eta^3}{\beta} \right)^{3/2} + \dots \right) \quad (12)$$

The constants a_1 therein have the following values

$$\begin{aligned}
 a_1 &= -\frac{2}{15} & a_5 &= 0.57627 \times 10^{-6} & a_9 &= 3.7282 \times 10^{-12} \\
 a_2 &= -\frac{1}{90} & a_6 &= 3.8907 \times 10^{-9} & a_{10} &= 2.2383 \times 10^{-13} \\
 a_3 &= \frac{-7}{(990 \times 15)} & a_7 &= -1.3986 \times 10^{-9} & a_{11} &= -0.3104 \times 10^{-14} \\
 a_4 &= 1.60333 \times 10^{-6} & a_8 &= -3.9135 \times 10^{-11} & a_{12} &= -1.081 \times 10^{-15}
 \end{aligned}$$

Due to the boundary condition at the wall, one integration constant is zero. The second integration constant is determined from the asymptotic behavior for large values of η . Because of $q_w(\psi, 0) = 0$ the first integral in equation (10) is eliminated. The second integral, however, yields by partial integration, with consideration of the asymptotic behavior of the error integral, just as in the case treated before by W. Wuest the solution

$$q_w B \sim \gamma \left[1 - \Phi \left(\frac{\psi}{\sqrt{4\gamma u_1 x}} \right) \right] \quad (13)$$

The constants β in equation (12) and γ in equation (13) are determined by the fact that for large η values q and $\frac{dq}{d\eta}$ according to equation (12) and equation (13) agree with each other. The recalculation of the two constants yielded the following values

$$\beta = 0.6642 \quad \gamma = 0.828$$

For comparison, L. Prandtl (reference 7) gives the following values calculated by Blasius and Tietjens which read, converted to the above designations

$$\beta = 2 \times 0.332 = 0.664$$

$$\gamma = 2\sqrt{\pi} 0.231 = 0.819$$

F. Riegels and J. A. Zaat give in a new report (reference 8) for γ the following value

$$\gamma = 0.342\sqrt{2\pi} = 0.857$$

The function q with first and second derivative has been tabulated and plotted in numerical table 1 and figure 2¹.

4. ASYMPTOTIC BEHAVIOR OF THE SUCTION BOUNDARY LAYER

For calculation of the asymptotic behavior of the suction boundary layer, we divide the function q_w defined by equation (8) into two parts

$$q_w = q_{wl} + q_{w2}$$

The first part is to be selected so that it satisfies the initial condition at the suction point $x = x_0$; this is done by extending the asymptotic solution of the Blasius boundary layer to $x > x_0$ as well. From equation (13) one then obtains

$$q_{wl} = \gamma \left[1 - \Phi \left(\frac{\psi + \psi_0}{\sqrt{4v u_1 x}} \right) \right]$$

¹The numerical table has been calculated with the values $\beta = 0.664$ and $\gamma = 0.819$.

Numerical Table 1. Blasius Boundary Layer

| η | $q(\eta)$ | $q'(\eta)$ | $q''(\eta)$ | $\sqrt{q(\eta)}$ |
|--------|-----------|------------|-------------|------------------|
| 0 | 0 | 0.6640 | 0 | 0 |
| .1 | .06606 | .6555 | .12751 | .2570 |
| .2 | .13106 | .6427 | .17750 | .3620 |
| .3 | .1939 | .6206 | .2114 | .4404 |
| .4 | .2546 | .5981 | .2369 | .5046 |
| .5 | .3135 | .5734 | .2561 | .5599 |
| .6 | .3695 | .5471 | .2700 | .6079 |
| .7 | .4228 | .5195 | .2796 | .6502 |
| .8 | .4681 | .4912 | .2856 | .6842 |
| .9 | .5211 | .4625 | .2883 | .7212 |
| 1.0 | .5659 | .4337 | .2883 | .7523 |
| 1.1 | .6081 | .4062 | .2857 | .7798 |
| 1.2 | .6469 | .3766 | .2809 | .8043 |
| 1.3 | .6831 | .3487 | .2743 | .8265 |
| 1.4 | .7167 | .3217 | .2660 | .8466 |
| 1.5 | .7474 | .2955 | .2563 | .8645 |
| 1.6 | .7758 | .2705 | .2457 | .8807 |
| 1.7 | .8017 | .2466 | .2340 | .8954 |
| 1.8 | .8252 | .2238 | .2248 | .9084 |
| 1.9 | .8465 | .2022 | .2088 | .9200 |
| 2.0 | .8657 | .1820 | .1955 | .9304 |
| 2.5 | .9352 | .1013 | .1303 | .9671 |
| 3.0 | .9715 | .0509 | .0774 | .9857 |
| 3.5 | .9881 | .0217 | .0381 | .9940 |
| 4.0 | .9962 | .0085 | .0169 | .9981 |
| 4.5 | .9988 | .0029 | .0066 | .9994 |
| 5.0 | .9997 | .0009 | .0022 | .9998 |

Therein $\psi = 0$ forms the new wall streamline and ψ_0 the suction quantity. The second part q_{w2} then must be chosen so that the boundary condition $q_w = q_w(0, x)$ is satisfied. If the asymptotic relation $q \sim 1 - q_w$ would rigorously apply in the entire domain of the boundary layer, there would have to be at the wall $q_w(0, x) = 1$, because of $q = 0$; however, the asymptotic solution deviates from the rigorous solution if it is continued up to the wall. Therefore $q_w(0, x)$ is an unknown function regarding which we merely make the assumption that it does not become infinite. As initial condition for the part q_{w2} one

further has $q_{w2}(\psi, x_0) = 0$ since $q_{w1}(\psi, x_0)$ already satisfies the initial condition

$$q_w = \gamma \left[1 - \Phi \left(\frac{\psi + \psi_0}{\sqrt{4\nu u_1 x_0}} \right) \right]$$

which insures the connection with the Blasius solution. The contribution q_{w2} to the solution also must obey the differential equation (9). In the solution (equation (10)) the first integral is eliminated, because of $q_{w2}(\psi, x_0) = 0$, whereas in the second integral one has to put

$$q_{w2}(0, x) = q_w(0, x) - q_{w1}(0, x) = q_w(0, x) - \gamma \left[1 - \Phi \left(\frac{\psi_0}{\sqrt{4\nu u_1 x}} \right) \right]$$

so that the asymptotic solution reads

$$q_w = \gamma \left[1 - \Phi \left(\frac{\psi + \psi_0}{\sqrt{4\nu u_1 x}} \right) \right] + \int_{x_0}^x \left\{ q_w(0, x') - \gamma \left[1 - \Phi \left(\frac{\psi_0}{\sqrt{4\nu u_1 x'}} \right) \right] \right\} \frac{\partial}{\partial x'} \Phi \left(\frac{\psi}{\sqrt{4\nu u_1 (x - x'_0)}} \right) dx'$$

By partial integration one obtains with consideration of the asymptotic behavior of the error integral (by W. Wuest, elsewhere)

$$q_w \sim \gamma \left[1 - \Phi \left(\frac{\psi + \psi_0}{\sqrt{4\nu u_1 x}} \right) \right] + \left\{ q_w(0, x_0) - \gamma \left[1 - \Phi \left(\frac{\psi_0}{\sqrt{4\nu u_1 x_0}} \right) \right] \right\} \left[1 - \Phi \left(\frac{\psi}{\sqrt{4\nu u_1 (x - x_0)}} \right) \right]$$

Because of the connection with the Blasius solution, however, $q_w(0, x_0) = \gamma$, if the asymptotic solution is continued up to the wall, so that one finally obtains as the asymptotic solution for the suction boundary layer

$$q_w \sim \gamma \left[1 - \Phi \left(\frac{\psi + \psi_0}{\sqrt{4\nu u_1 x}} \right) \right] + \gamma \Phi \left(\frac{\psi_0}{\sqrt{4\nu u_1 x_0}} \right) \left[1 - \Phi \left(\frac{\psi}{\sqrt{4\nu u_1 (x - x_0)}} \right) \right] \quad (14)$$

Instead of the error integrals Φ one may for large values of Ψ again go back to the Blasius solution if one takes the asymptotic behavior of the latter according to equation (8) and equation (13) into consideration

$$q \sim q_B \left(\frac{\Psi + \Psi_0}{\sqrt{\nu u_1 x}} \right) - \Phi \left(\frac{\Psi_0}{\sqrt{4 \nu u_1 x_0}} \right) \left[1 - q_B \left(\frac{\Psi}{\sqrt{\nu u_1 (x - x_0)}} \right) \right] \quad (15)$$

In this formula q_B represents the Blasius solution. The last form of the solution proves to be particularly expedient for the further considerations.

5. APPROXIMATE SOLUTION FOR THE SUCTION BOUNDARY LAYER

It suggests itself to generalize the asymptotic solution which is valid for large values of Ψ in the following manner

$$q = q_B \left(\frac{\Psi + \Psi_0}{\sqrt{\nu u_1 x}} \right) - F(\Psi, x) \left[1 - q_B \left(\frac{\Psi}{\sqrt{\nu u_1 (x - x_0)}} \right) \right] \quad (16)$$

Due to $q = 0$ for $\Psi = 0$ and because of equation (15) the function $F(\Psi, x)$ must satisfy the following conditions

$$F(0, x) = q_B \left(\frac{\Psi_0}{\sqrt{\nu u_1 x}} \right) \quad F(\infty, x) = \Phi \left(\frac{\Psi_0}{\sqrt{4 \nu u_1 x_0}} \right) \quad (17)$$

It was hoped at first that one could choose for F , as in the nonsteady analogue by W. Wuest, elsewhere correspondingly an exponential function as the simplest formulation; besides equation (17) the disappearance of the second derivative of q at the wall would be added as a further condition; however, it was shown that such a formulation does not meet with success and even, in a certain domain, does not yield any solution at all.

For the further calculation we introduce the following simplified notation

$$\frac{\psi + \psi_0}{\sqrt{v u_1 x}} = \eta \quad \frac{\psi}{\sqrt{v u_1 (x - x_0)}} = \eta' \quad \frac{\psi_0}{\sqrt{v u_1 x}} = \eta_0 \quad \eta = \eta_0 + \sqrt{\frac{x - x_0}{x}} \eta' \quad (18)$$

so that the solution (equation (16)) reads

$$q = q_B(\eta) - F \left[\left(1 - q_B(\eta') \right) \right] \quad (19)$$

According to a suggestion by A. Betz, we equate as first approximation q_1 the function F to the value dependent only on x

$$F_0(x) = F(0, x) = q_B \left(\frac{\psi_0}{\sqrt{v u_1 x}} \right) = q_B(\eta_0) \quad (20)$$

at the wall. Thus the first approximation reads

$$q_1 = q_B(\eta) - F_0 \left[\left(1 - q_B(\eta') \right) \right] \quad (21)$$

This formulation does not fulfill the boundary-layer equation (4) exactly. In particular, the second derivative of q_1 at the wall does not disappear; however, the dependency on the second derivative of the stability of the velocity profile is of a very sensitive nature so that one has to look for a more accurate solution. By substitution of the approximate solution (equation (21)) into the boundary-layer equation (4), one obtains

$$q_B''(\eta) \frac{x - x_0}{x} - \frac{\partial^2}{\partial \eta'^2} \left\{ F_0 \left[\left(1 - q_B(\eta') \right) \right] \right\} = 2(x - x_0) \frac{\partial}{\partial x} \sqrt{q_1} + \frac{\partial^2 \epsilon_1}{\partial \eta'^2}$$

Hence there results $\partial^2 \epsilon_1 / \partial \eta'^2$ as the error of this first approximation. By subtraction of the exact solution in which F stands for F_0 and q for q_1 , while ϵ_1 disappears, one then obtains

$$\frac{\partial^2}{\partial \eta'^2} \left\{ (F - F_0) \left[1 - q_B(\eta') \right] \right\} = \frac{\partial^2 \epsilon_1}{\partial \eta'^2} + \frac{\partial^2 \epsilon_2}{\partial \eta'^2} \quad (22)$$

where

$$\frac{\partial^2 \epsilon_2}{\partial \eta'^2} = \epsilon_2'' = 2(x - x_0) \frac{\partial}{\partial x} \left[\sqrt{q_1} - \sqrt{q} \right]$$

is an unknown function. The quantity ϵ_2'' disappears for $\eta' = 0$ and $\eta' = \infty$. By integration of equation (22) one obtains

$$(F - F_0) \left[1 - q_B(\eta') \right] = \epsilon_1 + \epsilon_2 \quad (23)$$

Therein ϵ_1 is to be determined graphically or numerically by repeated quadrature

$$\epsilon_1 = \int_{\infty}^{\eta'} \int_{\infty}^{\eta_2} \frac{\partial^2 \epsilon_1}{\partial \eta_1^2} d\eta_1 d\eta_2 \quad (24)$$

One may determine the asymptotic behavior of ϵ_2 by substituting in the above definition of ϵ_2'' for $\sqrt{q_1}$ and \sqrt{q} the asymptotic values $\sqrt{q_1} \sim 1 - \frac{1}{2} q_{w1}$ and $\sqrt{q} \sim 1 - \frac{1}{2} q_w$. Thereby one obtains

$$\epsilon_2'' \sim (x - x_0) \frac{\partial}{\partial x} (q_w - q_{w1})$$

Hence follows with use of equations (9), (19), (21), and repeated integration with respect to $\eta' = \psi / \sqrt{v u_1 (x - x_0)}$

$$\epsilon_2 \sim \gamma (F_{\infty} - F_0) \left[1 - \Phi \left(\frac{\eta'}{2} \right) \right] \quad (25)$$

As before, Φ denotes the error integral. Generally we visualize ϵ_2 as represented in the following manner

$$\epsilon_2 = \sum_{n=1}^{\infty} a_n(x) \left[1 - \Phi\left(\frac{\eta'}{2}\right) \right] \quad (26)$$

By way of approximation we limit ourselves to the first two terms, with $a_1 = \gamma(F_\infty - F_0)$ and a_2 determined by the fact that q must disappear at the wall. We determine accordingly the function F approximately to be

$$F = F_0 + \frac{1}{1 - q_B(\eta')} \left\{ \epsilon_1(\eta', x) + \gamma(F_\infty - F_0) \left[1 - \Phi\left(\frac{\eta'}{2}\right) \right] + a_2 \left[1 - \Phi'(\eta') \right] \right\} \quad (27)$$

$$a_2 = -\epsilon_1(0, x) - \gamma(F_\infty - F_0) \quad (28)$$

Calculation example. - $\psi_0/\sqrt{v u_1 x_0} = 0.125$ was selected as numerical example; F was calculated for the values $x/x_0 = 1.234, 1.562, 4.34$, and 9.78 and plotted in figure 3. For $x/x_0 = 1.562$ the error was determined by substitution of the approximated solution into the boundary-layer equation, and compared with the first approximation according to equation (21). Compared to the first approximation, a considerable improvement results particularly in the region near the wall (fig. 4). In figure 5 the results are converted to the velocity profile, in figure 6 the second derivative is represented. As a supplement, the connection between the degree of suction and the suction quantity of the magnitude $\eta_0^* = \psi_0/\sqrt{v u_1 x_0}$ will be supplemented. By the degree of suction Θ we here understand

$$\Theta = 1 - \frac{\delta_2^*}{\delta_1^*} \quad (29)$$

δ_1^* being the displacement thickness immediately ahead of the suction point and δ_2^* immediately behind it. Therewith Θ is given by

$$\Theta = \frac{\int_0^{\psi_0} \left(\frac{1}{\sqrt{q_B}} - 1 \right) d\psi}{\int_0^{\infty} \left(\frac{1}{\sqrt{q_B}} - 1 \right) d\psi} \quad (30)$$

The resulting values are tabulated in table 2 and plotted in figure 7.

Table 2. Degree of Suction

| η_0^* | 0 | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 2.0 |
|------------|---|-------|-------|-------|-------|-------|-------|-------|
| Θ | 0 | 0.392 | 0.520 | 0.671 | 0.762 | 0.824 | 0.870 | 0.974 |

The suction quantity ψ_0 is, furthermore, given by the following relation

$$\psi_0 = \sqrt{v u_1 x_0} \eta_0^* \quad (31)$$

6. SUMMARY

The development of a laminar boundary layer behind a suction point is investigated if by the suction merely the part of the boundary layer near the wall is "cut off", without the slot exerting a sink effect. As basis of the calculation, we used the boundary-layer equation in the form indicated by Prandtl-Mises which is closely related to the heat conduction equation or, respectively, to the differential equation of the nonsteady flow which is independent of the coordinate x along the wall. With consideration of the asymptotic behavior of the solution, an approximate solution is developed which is similar in structure to the solution of the nonsteady analogue which has been treated in an earlier report by W. Wuest, elsewhere.

Translated by Mary L. Mahler
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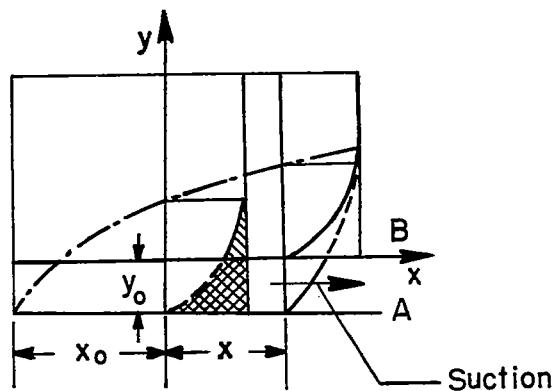


Figure 1.- Boundary-layer suction at the flat plate without sink effect.

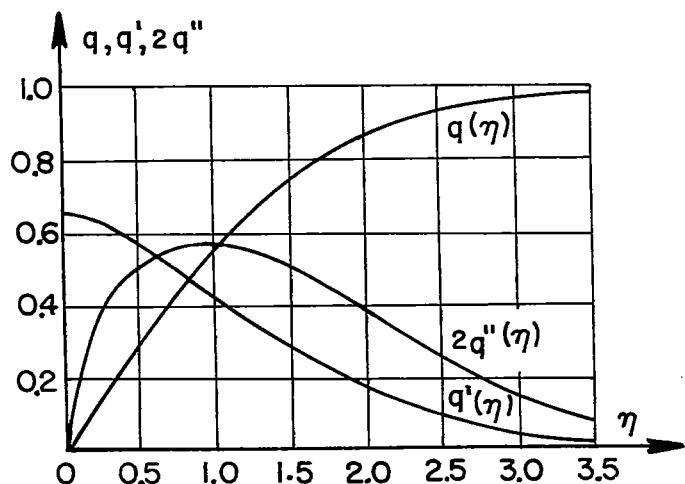


Figure 2.- The function $q(\eta)$ of the Blasius boundary layer with first and second derivative.

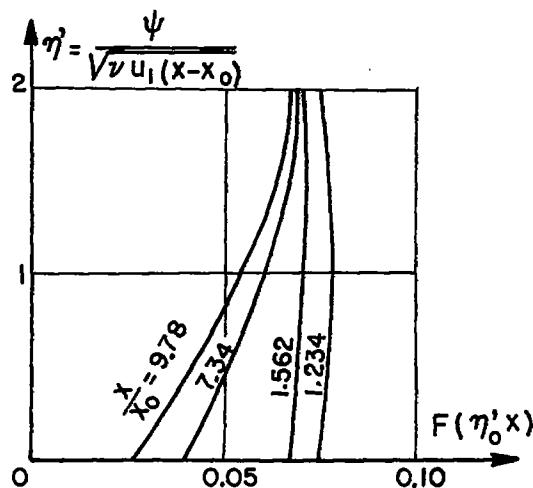


Figure 3.- Auxiliary function $F(\eta', x)$ for calculation of the suction boundary layer for $\psi_0 / \sqrt{\nu u_1 x_0} = 0.125$.

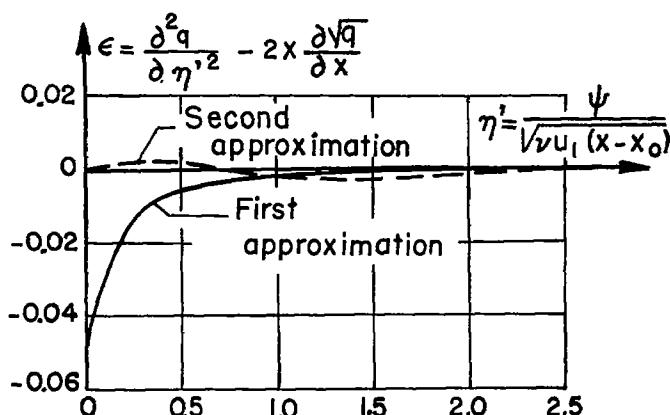


Figure 4.- Error of the first and second approximation for $x/x_0 = 1.562$.

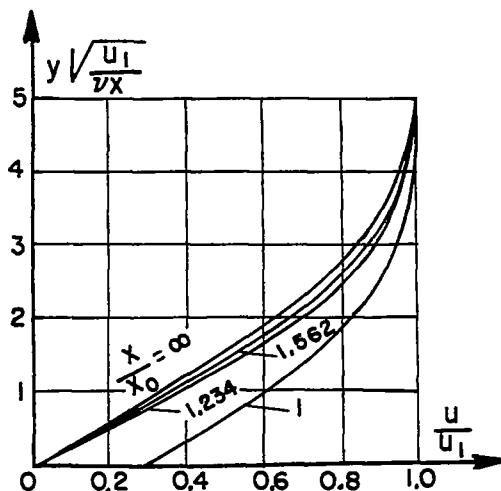


Figure 5.- Velocity profiles of the suction boundary layer for $\psi_0/\sqrt{v u_1 x_0} = 0.125$ and various distances from the suction point.

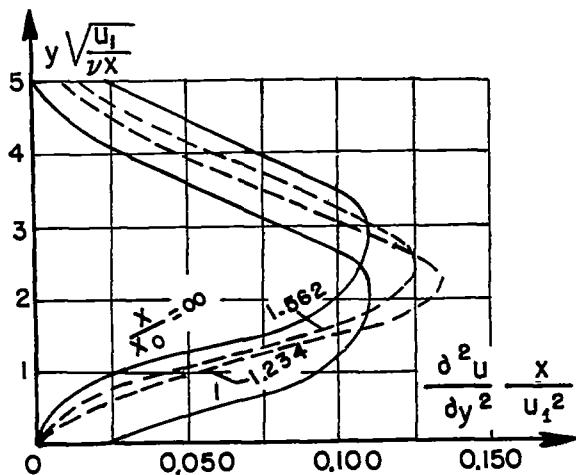


Figure 6.- Second derivative of the velocity profiles of the suction boundary layer for $\psi_0/\sqrt{v u_1 x_0} = 0.125$.

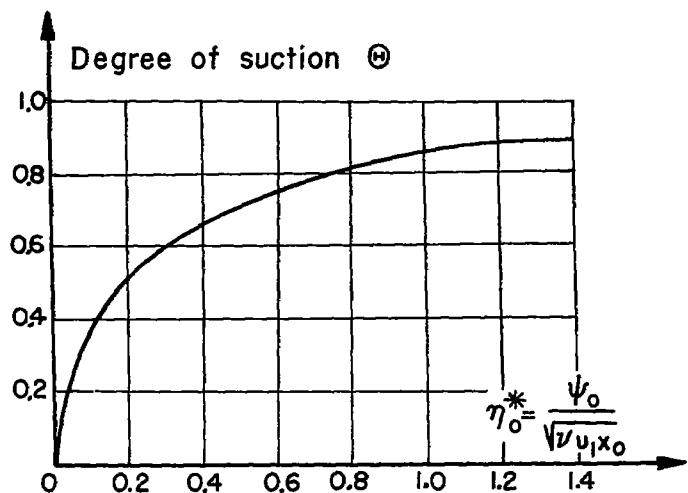


Figure 7.- Degree of suction (ratio of the cross-hatched and the total shaded area in fig. 1).